

# Amino acid positions subject to multiple coevolutionary constraints can be robustly identified by their eigenvector network centrality scores

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## ABSTRACT

As proteins evolve, amino acid positions key to protein structure or function are subject to mutational constraints. These positions can be detected by analyzing sequence families for amino acid conservation or for coevolution between pairs of positions. Coevolutionary scores are usually rank-ordered and thresholded to reveal the top pairwise scores, but they also can be treated as weighted networks. Here, we used network analyses to bypass a major complication of coevolution studies: For a given sequence alignment, alternative algorithms usually identify different, top pairwise scores. We reconciled results from five commonly-used, mathematically divergent algorithms (ELSC, McBASC, OMES, SCA, and ZNMI), using the LacI/GalR and 1,6-bisphosphate aldolase protein families as models. Calculations used unthresholded coevolution scores from which column-specific properties such as sequence entropy and random noise were subtracted; “central” positions were identified by calculating various network centrality scores. When compared among algorithms, network centrality methods, particularly eigenvector centrality, showed markedly better agreement than comparisons of the top pairwise scores. Positions with large centrality scores occurred at key structural locations and/or were functionally sensitive to mutations. Further, the top central positions often differed from those with top pairwise coevolution scores: instead of a few strong scores, central positions often had multiple, moderate scores. We conclude that eigenvector centrality calculations reveal a robust evolutionary pattern of constraints—detectable by divergent algorithms—that occur at key protein locations. Finally, we discuss the fact that multiple patterns coexist in evolutionary data that, together, give rise to emergent protein functions.

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**Key words:** protein evolution; coevolution; amino acid; sequence alignment; graph theory; LacI/GalR; aldolase.

## INTRODUCTION

On-going genomic sequencing has generated a huge number of protein sequences. To place them into biological context, sequences are usually grouped into protein families that, in turn, can be used to reveal sequence/structure/function relationships. A common approach is to identify which amino acid positions are constrained during evolution. These positions are presumably crucial for the protein’s structure or function, and their mutation can provide key insights to a protein’s function. Various algorithms have been devised to detect positions that are conserved across a protein family or that vary among alternative lineages.<sup>1–13</sup> In addition, constraints on corresponding pairs of positions can be detected using coevolution analyses.

Coevolutionary algorithms seek to identify pairs of positions that vary together across evolutionary time. For example, if an amino acid change at position X correlates with a change at position Y, positions X and Y are said to coevolve. Coevolution suggests that the two positions

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are linked to carry out important structural or functional roles. Such patterns can arise from short-range biophysical constraints (such as charge–charge interactions) but are also commonly observed between positions that are distant from each other on the protein structure, via mechanisms that are not yet fully understood.<sup>14–16</sup>

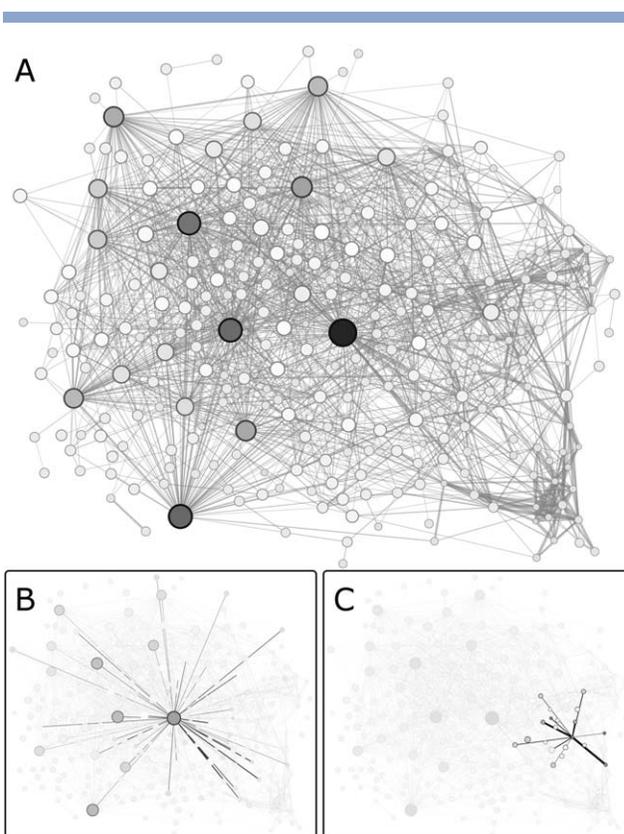
Several algorithms, with a range of mathematical foundations, have been devised to quantify coevolutionary patterns in multiple sequence alignments (MSAs).<sup>17–27</sup> These algorithms determine pairwise scores for all possible combinations of amino acid positions; the top pairwise scores indicate the strongest pattern of covariation. Despite searching for a common pattern, results from alternative algorithms seldom identify the same top pairwise scores.<sup>23,28,29</sup> One explanation is that some algorithms have flawed predictions. On the other hand, alternative algorithms might detect distinct evolutionary signatures, each biologically important. To date, mutagenesis experiments have not identified any single algorithm as better than the others for identifying positions with coevolutionary, mutational constraints.<sup>†</sup>

We hypothesized that deep patterns in coevolutionary data might be robust to algorithmic details and therefore detectable by multiple algorithms. To that end, we treated unthresholded coevolutionary scores as weighted networks (complete graphs) and used network centrality measures to identify important nodes. This allowed us to determine whether central nodes (positions) were more robustly identified by network analysis than by specific edges connecting a pair of positions.

To identify the central positions, we used the pairwise coevolutionary scores as edges of a weighted complete graph to calculate two network centrality scores: degree centrality<sup>30</sup> (DC) and eigenvector centrality<sup>31</sup> (EVC). For each amino acid position (node), DC simply takes the sum of connecting edge weights, whereas EVC takes into account both the weight of its connecting edges and the centrality of its connected partners. In practice, we found little difference between these metrics; since EVC has an established history in other coevolution studies, we focused downstream analyses on these results.

Centrality scores can be rank-ordered to reveal the most central positions of the coevolution network. In analyzing these scores, we first considered whether the central nodes (i) were identical to positions with the highest pairwise scores, or (ii) arose from multiple, intermediate scores (Fig. 1), corresponding to multiple coevolutionary constraints from other amino acid positions. Indeed, for the protein families of this study, positions with high EVC scores were frequently distinct from positions with the highest pairwise coevolutionary scores. Furthermore, when results were compared among five alternative coevolutionary algorithms, the EVC positions showed better agreement than the top pairwise positions. Comparison with experimental

<sup>†</sup>Some coevolutionary algorithms and methods for constructing MSAs do appear to out-perform others in predicting direct structural contacts. However, as we noted above, this is not the sole determinant of algorithm success; long range coevolution can also be biologically significant.



**Figure 1**

Network representation of coevolutionary scores for the LacI/GalR family. Note that these networks depict coevolutionary scores, *not* structural contacts. (A) In this example, the ZNMI algorithm was used to calculate pairwise coevolution scores. High scores are represented as thick edges; weak scores are represented with thin edges. Each node corresponds to an MSA column (amino acid position); node sizes and colors are scaled according to EVC scores (large, black = high, small, white = low). For figure clarity, only the top ~4% of edges are shown; however, all edge weights (coevolution scores) were used for EVC calculations. From the network shown in A, individual nodes and their edges are highlighted in panels (B) and (C), to illustrate the difference between a top EVC position with many moderate scores (B) and a position with a high, pairwise coevolution score but a low EVC score (C).

results showed that positions with high EVC scores have key structural and functional roles in the LacI/GalR and aldolase protein families. Thus, EVC calculations detected a robust evolutionary pattern of amino acid changes at key protein locations and can be used to guide experimental studies of protein function. Finally, we discuss the existence and interplay of multiple patterns in evolutionary data that, together, give rise to emergent protein functions.

## MATERIALS AND METHODS

### Protein families, sequence alignments, and MARS software

The MSA for the LacI/GalR family was constructed in 2011<sup>32</sup> and further refined in 2013.<sup>28</sup> This MSA

contains 351 representative sequences from 34 ortholog groups; the full sequence set (>2000 sequences) is too large for many calculations. Sequences for the aldolase MSA were obtained by iterative rounds of PSI-BLAST,<sup>33</sup> and manually edited to remove redundant sequences. To take advantage of available sequence and structural data, a representative subset was aligned using PROMALS3D.<sup>34</sup> The remaining sequences were clustered based on their sequence identity into groups containing at least one of the “representative” sequences in the PROMALS3D alignment. These high sequence identity clusters were re-aligned with MUSCLE.<sup>35</sup> Group alignments were integrated into the full aldolase alignment with the custom software MARS-Prot (Supporting Information, Supplementary Methods) using the representative sequences as guides.

The MARS-Prot software is a general tool for integrating new sequences into existing sequence alignments that does not perturb the labor-intensive editing that is required to produce high quality MSAs. This tool is greatly needed: extensive ongoing genomic sequencing requires frequent updates to existing MSAs. The MARS-Prot algorithm is described in Supporting Information, Supplementary Methods; references cited therein are cited here.<sup>36,37</sup> MARS-Prot is freely available under an open source license (<https://github.com/djparente/MARS>).

The LacI/GalR and aldolase protein alignments used in this study are available upon request. As a final check, we considered the number of sequences and phylogenetic sampling in these MSAs, which has been shown to affect MSA analyses.<sup>38,39</sup> For example, coevolution analyses might return biased results if one lineage of the family were oversampled. To check for this possibility, the sequence identity matrix for each MSA was manually inspected to ensure that no group of highly similar sequences was over-represented relative to the rest of the MSA. In addition, maximum-likelihood phylogenetic trees were inferred using RAxML 7.0.<sup>40</sup> with default parameters under the PROTGAMMABLOSUM62 substitution model. Trees were visualized with PhyloWidget<sup>41</sup> (Supporting Information Fig. S1). Trees for both families have a stellate appearance, indicating that many lineages are represented and that no one lineage dominated the calculation.<sup>38</sup>

## Coevolution analyses

Coevolution scores were calculated using five alternative algorithms: Explicit Likelihood of Subset Covariation (ELSC<sup>22</sup>), Observed Minus Expected Squared (OMES<sup>21,29</sup>), McLachlan-based Substitution Correlation (McBASC<sup>17,18,20</sup>), Statistical Coupling Analysis (SCA<sup>19</sup>), and Z-Normalized Mutual Information (ZNMI<sup>23</sup>). For ELSC, OMES, McBASC and SCA, coevolution scores were calculated using a software implementation by Fodor *et al.*<sup>22,29</sup> For ZNMI, scores were calculated using our re-implementation of that method.<sup>28</sup> For all algorithms, positions with >50% gaps or very high conservation (<5%

sequence variability, which corresponds to sequence entropy < 0.1985<sup>23</sup>), were excluded from further analysis. For ZNMI, positions with >10% gaps were further excluded, to match its initial implementation by Brown and Brown.<sup>23</sup>

All analyses were carried out using an ensemble-based approach.<sup>23,28</sup> For the LacI/GalR MSA, 100 sub-alignments were constructed, each including 90% of the available sequences. For each sub-alignment, coevolution scores were calculated. For each position, the scores from the 100 ensembles were averaged; these are reported as the “initial” coevolutionary scores. The ensemble average approach limits the impact that might arise if a few sequences are misaligned. As an additional control, we generated a second ensemble containing 50% of the sequences and calculated its average coevolution scores. If the sequences in the starting MSA are well sampled, we expect that results for the 50% ensemble will agree with those of the 90% ensemble (Supporting Information Table S1).

Note that the ensemble/average approach should only be used to supplement direct inspection of the sequences included in an MSA. In other analyses, we showed that this control can fail in cases of extreme over-representation, because even randomly deleting 50% of the sequences leaves a sufficient number of redundant sequences to bias the calculations. For example, 75 LacI/GalR sequences fall in the LacI subfamily; their sequence identities ranged from 37% to 99% and the subfamily phylogenetic tree had nine major branches. When these subfamily sequences were used in coevolutionary analyses, results for the 50 and 90% ensembles were in good agreement. However, for both the pairwise and the network centrality scores, the top 20 consensus positions unexpectedly included many positions that tolerate multiple substitutions (Supporting Information Table S2), which indicates that they are not functionally important. Manual inspection of the LacI subfamily revealed that 41% of the sequences were >95% identical to each other. Even with a randomized, 50% sampling rate, these over-represented sequences are likely to dominate the coevolutionary calculation, leading to the misleading results. This example shows that computed controls do not substitute for manual inspection of an MSA. Note that neither the LacI/GalR nor the aldolase families had over-represented sequences.

Like the LacI/GalR family, the aldolase family (1562 sequences) was too large for some calculations. Therefore, we constructed an ensemble of aldolase alignments, each with 500 sequences as the “90%” ensemble. A second ensemble of alignments containing 278 sequences was used for the “50%” alignment.

For both the LacI/GalR and aldolase families, the 90 and 50% scores had good agreement, which indicates that a sufficient number of sequences were included in each alignment. (Supporting Information Fig. S2, Supporting Information Table S1). Further analyses, therefore, focused on only the 90% ensemble.

Next, we considered that alternative coevolutionary algorithms have divergent responses to various properties of MSA columns (such as sequence entropy or random noise) which might introduce algorithm-specific noise.<sup>29</sup> To account for these contributions, we created shuffled alignments by independently randomizing the order of amino acids contained within each column. This maintains column properties such as amino acid distribution and sequence entropy, but destroys both phylogenetic patterns of evolutionary change and coevolutionary relationships between pairs of positions. Shuffled alignments have been used as benchmarks for “sector” analyses,<sup>42</sup> and various subtractions have been carried out for protein–protein coevolution.<sup>43,44</sup> However, to the best of our knowledge, this is the first time that shuffled alignments have been used to estimate spurious signals arising in intra-protein covariation analyses. To that end, we performed “coevolutionary” calculations on each shuffled MSA. Then, for each pair of positions, the score from the shuffled MSA was subtracted from the score of the unshuffled MSA. This calculation is a linear approximation: it assumes that initial (unshuffled) coevolution scores were the sum of the true signals and noise. We refer to the resulting coevolution scores as “subtracted.”

To determine consensus coevolution scores for the initial and subtracted data sets, we used the method of Parente *et al.*,<sup>28</sup> with the variation of using the median Z-score, rather than the mean, to rank-order positions. (For more details, see consensus EVC scores, below.)

### Z-normalized decoy adjusted mutual information

In subtracting the MSA noise component to reveal “true” coevolutionary components, we assumed that the magnitude of the noise was the same in the original and shuffled alignments. This assumption should hold for ELSC, McBASC, OMES and SCA, but is violated by ZNMI. For a pair of positions  $x$  and  $y$ , ZNMI scores are calculated in a three-step process<sup>23</sup>: (1) the mutual information,  $MI(x,y)$ , is estimated; (2)  $MI(x,y)$  is divided by the joint sequence entropy of positions  $x$  and  $y$  to calculate the normalized mutual information,  $NMI(x,y)$ ; and (3) the  $NMI(x,y)$  scores are further z-normalized against the joint distribution of  $NMI(x,y)$  scores for positions  $x$  and  $y$ . Thus, the third step of this procedure scales the joint distribution of ZNMI( $x,y$ ) scores to a normal distribution with mean of 0 and variance of 1. An assumption of this procedure is that there are true coevolutionary signals to detect: positions with high ZNMI scores are those that coevolve strongly, relative to each position’s own NMI distribution. However, in shuffled alignments, when coevolutionary signals have been destroyed, the ZNMI distribution is still scaled to have a normal distribution with mean 0 and variance 1. Thus, the Z-normalization step of ZNMI would spuriously scale the noise of the shuffled alignments to the same level as the coevolutionary signal in the original alignments.

Subtracting away the shuffled ZNMI score would therefore amplify, rather than eliminate, the influence of noise.

To prevent this undesirable outcome, we performed score subtraction before the Z-normalization step, by subtracting NMI of each shuffled score from the original score. This variant of the ZNMI algorithm we call the Z-Normalized Decoy Adjusted Mutual Information (ZNDAMI), mathematically defined as:

$$\text{ZNDAMI}(x, y) = \frac{\text{NMI}_{\text{sub}}(x, y) - \mu_{xy}}{\sigma_{xy}} \quad (1)$$

where  $\text{NMI}_{\text{sub}}(x,y)$  is the subtracted NMI score<sup>‡</sup> (i.e.,  $\text{NMI}_{\text{orig}}(x,y) - \text{NMI}_{\text{shuf}}(x,y)$ ) and  $\mu_{xy}$  and  $\sigma_{xy}$  are the mean and standard deviation of the joint  $\text{NMI}_{\text{sub}}$  distribution for positions  $x$  and  $y$ , namely:

$$\mu_{xy} = \frac{\mu_x \sigma_y^2 + \mu_y \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \quad (2)$$

and

$$\sigma_{xy}^2 = \frac{\sigma_x^2 \sigma_y^2}{\sigma_x^2 + \sigma_y^2} \quad (3)$$

which depend on the individual distribution mean:

$$\mu_x = \frac{1}{N} \sum_z \text{NMI}_{\text{sub}}(x, z) \quad (4)$$

and variance:

$$\sigma_x^2 = \frac{1}{N-1} \sum_z (\text{NMI}_{\text{sub}}(x, z) - \mu_x)^2 \quad (5)$$

where  $N$  is the number of columns in the alignment that meet the gap criterion (<10% gaps, see Methods).

### Eigenvector network centrality

To identify central nodes in the ELSC, McBASC, OMES, SCA, and ZNMDAMI coevolution networks, we calculated the eigenvector centrality<sup>31</sup> for each position in the MSA. These calculations were performed on both the initial (unsubtracted) and subtracted pairwise coevolutionary scores (see above). Eigenvector centrality calculations take into account both the strength of connectivity and the centrality of strongly connected partners, and has been utilized for many diverse applications, including: internet search engines,<sup>45,46</sup> social network analysis,<sup>47</sup> political analysis,<sup>48</sup> fMRI data processing,<sup>49</sup> and epidemiological disease-transmission networks.<sup>50,51</sup>

<sup>‡</sup>As implemented, the  $\text{NMI}_{\text{sub}}$  is actually the original NMI score minus the average NMI score in an ensemble of 10 reshuffled alignments.

A key feature of our calculation is that we did *not* impose a significance threshold on the pairwise coevolutionary data prior to EVC calculations. Although eventually necessary, we deferred thresholding until the end of analyses, which allowed us to detect positions with high EVC scores that would have otherwise been missed (Supporting Information Table S3 and Supporting Information Fig. S3). The eigenvector centrality score of the  $i$ 'th position is equal to the  $i$ 'th row of the dominant column eigenvector of the adjacency matrix. That is, for node  $s_i$ , we take

$$s_i = \frac{1}{\lambda} \sum_{j=1}^n W_{ij} s_j$$

where  $W_{ij}$  are entries of  $\mathbf{W}$ , the weighted adjacency matrix, corresponding to the weight between nodes  $s_i$  and  $s_j$ . This can be rewritten

$$\lambda \mathbf{s} = \mathbf{W} \cdot \mathbf{s} \quad (7)$$

where  $\mathbf{s}$  is the eigenvector we seek, corresponding to the (necessarily) unique largest eigenvalue  $\lambda$  as long as the centralities are non-negative.<sup>52</sup> Eigenvector centrality calculations were carried out using the NetworkX Python package (<http://networkx.github.io/>).

In practice, eigenvector centrality is computed iteratively, as follows: every node is initialized with the same starting score. Then, at subsequent iterations, the score of a node is (i) set equal to the edge-weighted sum of the current score of its neighbors, and (ii) normalized so that the squared sum of the EVC scores is constant. This process is iterated to convergence to produce a final eigenvector centrality score for each position. Informally, this procedure is analogous to first initializing every node to be equally important, reflecting our (initial) ignorance as to which nodes were more central. At every step, nodes that are strongly connected to other important nodes are *preferentially* selected to gain disproportionately more “importance,” allowing the iteration to uncover important nodes.

Eigenvector centrality calculations were carried out using the NetworkX python package (<http://networkx.github.io/>). More formally, the eigenvector centrality score of position  $i$  in the  $T$ 'th iteration ( $s_{i,T}$ ), in a network of  $N$  positions, is given by the iterative system:

$$s_{i,T+1} = \frac{q_{i,T}}{\sqrt{\sum_j (q_{j,T})^2}} \quad (8)$$

$$q_{i,T} = \sum_j (w_{i,j}) (s_{j,T}) \quad (9)$$

$$s_{i,0} = 1/\sqrt{N} \quad (10)$$

where  $w_{i,j}$  is the weight of the edges between positions  $i$  and  $j$ . For the centrality score to converge, scores must be non-negative. This is true for all algorithms except ZNMI, which transforms scores to be z-normalized with

a mean of zero. Thus, for all algorithms, scores were linearly transformed to fall in the range [0,1]. The final eigenvector centrality score assigned to position  $i$ ,  $s_i$ , is given by:

$$s_i = \lim_{T \rightarrow \infty} s_{i,T} \quad (11)$$

This procedure is identical to an algorithm called “power iteration” that calculates the dominant eigenvector of a matrix. For networks of coevolutionary scores, power iteration is applied to the adjacency matrix of the network.

### Network degree centrality

We compared EVC scores to a simpler network centrality score, degree centrality<sup>30</sup> (Supporting Information Figs. S4 and S5 and Supporting Information Table S4). This score computes the total weight of edges directly connected to each node. As with the EVC scores, weights were transformed to fall on the interval [0,1] for the calculation.

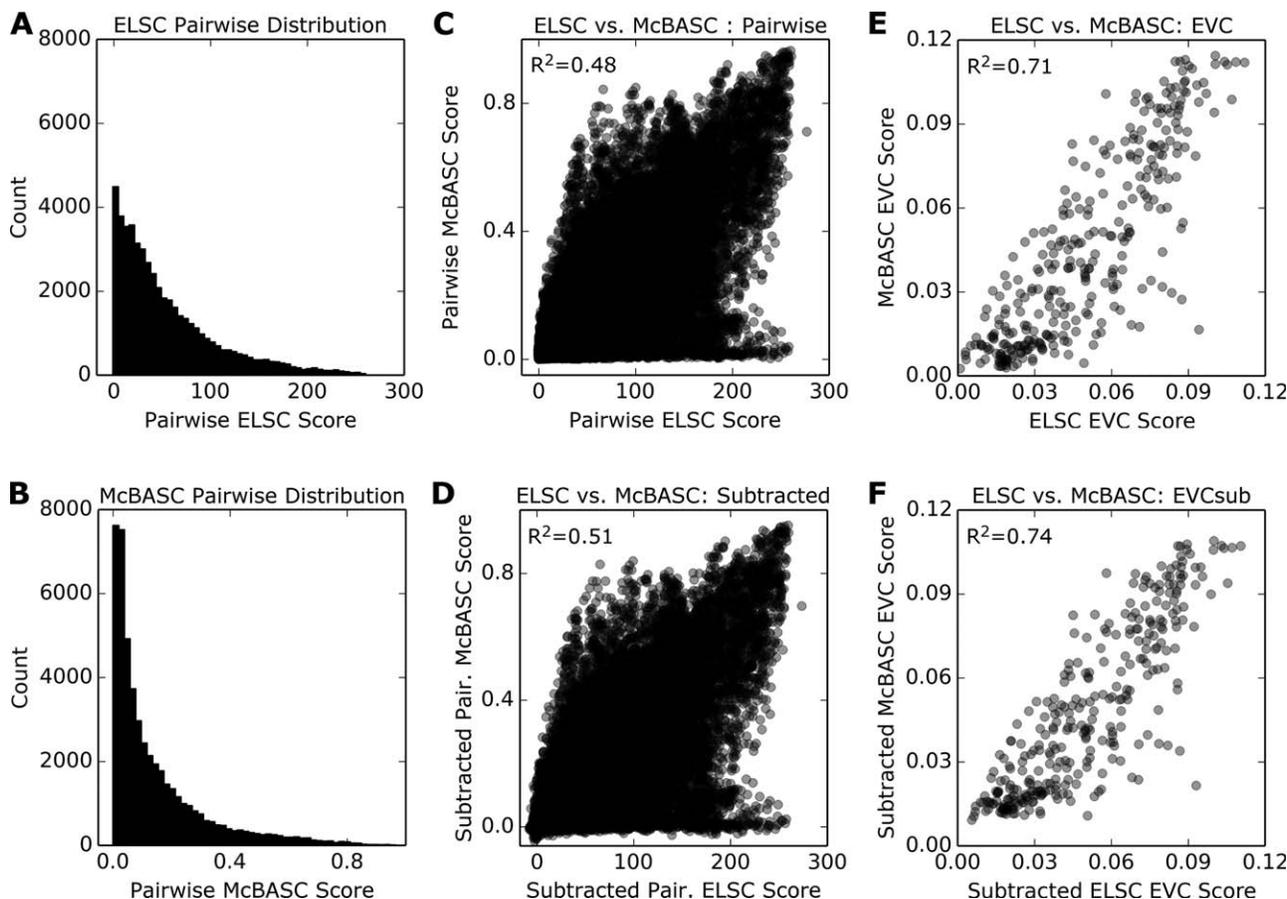
### Network comparisons and consensus

In comparing various sets of results (e.g., initial versus subtracted pairwise scores, or pairwise versus EVC scores), we had no a priori reason to assume that scores should be linearly related. Thus, we used a nonparametric measure—Spearman  $R^2$ —which can detect any monotonic relationship between two variables. Similarity was quantified by calculating Spearman  $R^2$  for the edge weights assigned to each pair of positions in the network. This parameter ranges from 0.0–1.0.

For comparing EVC scores to experimental data, we determined consensus scores using results from all 5 algorithms for each position. Since each coevolution algorithm uses a different output scale [e.g., Fig. 2(A,B)], each set of scores was first Z-normalized to standardize the mean and variance values. This prevents one algorithm from dominating downstream calculations. The Z-normalized EVC scores of the 5 algorithms were then used to determine a median EVC score for each position. Results were compared to crystal structures from the aldolase (PDB: 1xfb) and LacI/GalR families (PDBs: 1efa, 1wet, 1rzr, 2nzv, 3oqo, and 1byk).<sup>53–59</sup> Molecular graphics were created using UCSF Chimera 1.8.<sup>60</sup>

### TEA-O analyses

To determine which amino acid changes in each MSA track with phylogeny, we used TEA-O analyses (<http://nava.liacs.nl/kye/TEA-O/>).<sup>9</sup> Results for the LacI/GalR family were taken from Tungtur *et al.*<sup>32</sup> These results were shown to be independent of the method used for generating a phylogenetic tree. Similar analyses were performed for the aldolase family, using the phylogenetic tree shown in Supporting Information Figure S1.

**Figure 2**

Eigenvector centrality scores show better correlation between coevolution algorithms than do pairwise scores. The example shown is for the aldolase family. Histograms show the distribution of coevolutionary scores from (A) ELSC and (B) McBASC. Between these two algorithms, correlation of (C) initial (unsubtracted) or (D) subtracted pairwise coevolution scores have modest correlations, as indicated by their Spearman  $R^2$  values. However, correlation of (E) unsubtracted or (F) subtracted EVC scores is greatly improved, as indicated by larger  $R^2$  values.

## RESULTS

For this study, we used the LacI/GalR transcription regulators and the family of 1,6-bisphosphate aldolase (“aldolase”) (Supporting Information Table S1). The LacI/GalR family comprises bacterial paralogs with sequence identities that range from  $\sim 19$ – $99\%$ .<sup>32</sup> The aldolase family was chosen as another family with highly divergent sequence identities ( $\sim 19$ – $99\%$ ), but comprising orthologs instead of paralogs. These “class I” aldolase homologs are found in all animals, plants, and green algae.<sup>61</sup>

### Analyses of coevolutionary networks

Coevolution analyses generate a large number of pairwise scores ( $N^2$ , where  $N$  is the number of columns of amino acid positions). Most often, these scores have been rank-ordered to identify the top pairwise scores or organized into an all-vs-all heatmap (e.g., Ref. 42). Alternatively, coevolution scores have been recast as networks.

For example, several studies imposed a threshold (such as  $Z \geq 4$ ) on the pairwise coevolution scores and created networks in which top-scoring positions correspond to nodes and coevolution scores correspond to edges.<sup>62–65</sup> A useful depiction of thresholded networks is the “circos” plot.<sup>27</sup> Recently, we used all coevolution scores to weight the edges between all nodes to directly compare outcomes for related protein families.<sup>28</sup> Here, we have analyzed pairwise coevolution scores as unthresholded networks to identify features that are robustly identified by mathematically-divergent algorithms.

To identify robust features, each MSA of this study was analyzed using five common coevolution algorithms with diverse mathematical foundations: (i) OMES measures a  $\chi^2$ -like goodness-of-fit parameter<sup>21,29</sup>; (ii) McBASC generalizes the correlation coefficient to categorical data<sup>17,18,20</sup>; (iii) ELSC<sup>22</sup>; and (iv) SCA<sup>19</sup> take a perturbative approach; and (v) ZNMI<sup>23</sup> uses an information theoretic approach to measure shared

**Table I**  
Eigenvector Centrality Reconciles Results for Various Coevolution Analyses

Algorithms			Coefficient of determination (Spearman $R^2$ )							
			Aldolase				LacI/GalR			
			Pairwise		EVC		Pairwise		EVC	
			Initial	Sub	Initial	Sub	Initial	Sub	Initial	Sub
ELSC	vs.	McBASC	0.48	0.51	0.71	0.74	0.09	0.11	0.27	0.50
ELSC	vs.	OMES	0.64	0.65	0.78	0.78	0.37	0.42	0.66	0.73
ELSC	vs.	SCA	0.19	0.23	0.40	0.43	0.02	0.14	0.14	0.32
ELSC	vs.	ZNMI	0.44	0.46	0.78	0.79	0.23	0.25	0.45	0.60
ELSC	vs.	TEA-0				0.30				0.54
McBASC	vs.	OMES	0.59	0.63	0.72	0.75	0.14	0.20	0.14	0.38
McBASC	vs.	SCA	0.20	0.23	0.22	0.26	0.08	0.13	0.12	0.36
McBASC	vs.	ZNMI	0.48	0.52	0.65	0.69	0.18	0.28	0.05	0.34
McBASC	vs.	TEA-0				0.26				0.38
OMES	vs.	SCA	0.25	0.25	0.31	0.31	0.31	0.34	0.43	0.55
OMES	vs.	ZNMI	0.50	0.54	0.77	0.85	0.46	0.50	0.51	0.69
OMES	vs.	tEA-0				0.26				0.70
SCA	vs.	ZNMI	0.35	0.37	0.60	0.61	0.18	0.18	0.33	0.47
SCA	vs.	TEA-0				0.25				0.42
ZNMI	vs.	TEA-0				0.34				0.77
Median improvement				0.03	0.19	0.23 <sup>a</sup>	0.05	0.12	0.29 <sup>a</sup>	

For each pair of algorithms, non-parametric correlation coefficients (Spearman  $R^2$ ) are shown for pairwise coevolution and eigenvector (EVC) scores, using either the unsubtracted (“Initial”) or subtracted (“Sub”) networks. As discussed in Methods, ZNDAMI is used in place of subtracted ZNMI.

<sup>a</sup>Median improvement does not include the TEA-O comparisons.

information content. As expected, for all MSAs, the five algorithms generated different rank orders for the unsubtracted pairwise scores [Table I; Fig. 1(C) and Supporting Information Figs. S6–S9]. Pairs that were assigned a high coevolution score by one algorithm were often assigned moderate or low scores by another. When only the top score was considered for each position (in network formalism, the maximum edge weight, “MEW”), algorithm agreement did not improve (Supporting Information Table S5).

We next attempted to reconcile results by correcting for the different algorithm sensitivities to non-coevolutionary signals, such as sequence entropy and random noise.<sup>29</sup> To estimate these spurious signals, we created “shuffled” alignments by separately randomizing the amino acids within each column of the LacI/GalR and aldolase MSAs. This maintains column properties such as amino acid distribution and sequence entropy, but destroys both phylogenetic patterns between and coevolutionary relationships within naturally evolved sequences. All coevolutionary calculations were repeated for the shuffled alignments, and for each pair of positions in the coevolution network, the shuffled scores were subtracted from those of the unshuffled MSAs. When the final scores were compared, the subtraction process only modestly improved comparisons of pairwise scores between alternative algorithms (Table I). However, as discussed below, subtraction improved centrality scores to a greater degree.

Notably, when comparing the pairwise scores, no single algorithm appeared to perform “better” than the

others. A criterion frequently used to indicate algorithm success is mutational sensitivity of top scoring positions. For the LacI/GalR family, most of the positions with top, subtracted pairwise scores are sensitive to mutagenesis, regardless of the algorithm (Supporting Information Fig. S10).

Thus, we were curious whether deeper patterns in coevolutionary data would also identify functionally important positions, and whether they would be more robustly detected by multiple algorithms. This required that we first consider how and when coevolution scores are thresholded: Like most other metrics of evolutionary patterns in MSAs, coevolution scores are continuous, with no clear breaks to separate “important” from “not important.” Threshold choice is always arbitrary: Conservative thresholds eliminate meaningful data; liberal thresholds may include data with no meaning. In this work, we deferred thresholding as long as possible. We chose to analyze coevolution scores as unthresholded, weighted networks [e.g., Fig. 1(A,B)], using graph theory to calculate network centrality.

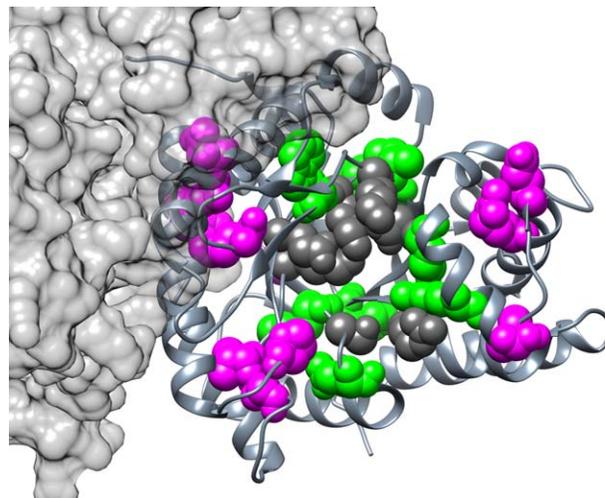
Network centrality can be calculated in a number of different ways.<sup>66</sup> Most existing methods were developed for social network analyses. Methods such as “closeness” and “betweenness” centrality measure the number of steps between nodes,<sup>66</sup> and thus do not make sense for the complete graphs created by unthresholded, pairwise coevolution scores. In contrast, both degree (DC)<sup>30</sup> and eigenvector (EVC)<sup>31</sup> centrality measure the effect of one node on all other nodes, based on the weights of the connecting edges. Whereas DC simply sums the weight

of edges connected to each node, EVC accounts for the centrality of neighboring nodes as well. DC and EVC might identify the same nodes, but in principle EVC is sensitive to more subtle network effects. In the current study, DC results agreed strongly with EVC results (Supporting Information Figs. S4 and S5). For simplicity, we focused remaining analyses on EVC results.

We first compared that the top EVC scores to the top pairwise coevolutionary scores (MEW, defined above). Notably, the MEW and EVC scores do not consistently correlate, by either Spearman correlation coefficients (Supporting Information Table S5) or by Jaccard indices (Supporting Information Fig. S3). Thus, we conclude that EVC calculations for the LacI/GalR and aldolase families identify a group of amino acid positions that is distinct from the pairwise positions. That is, the centrality calculations can discriminate positions with many moderate scores from those with one strong but many weak scores [e.g., Fig. 1(B) vs. Fig. 1(C)]. Furthermore, if pairwise coevolution scores were thresholded (limited to the top scores) prior to downstream analyses, several positions with high EVC scores would not be detected.

Next, we compared the EVC scores calculated by the alternative coevolutionary algorithms. Agreement among EVC scores was greatly improved over all comparisons of pairwise scores, and across the whole range of scores. For the EVC scores determined for the initial (unsubtracted) networks, almost all comparisons showed better correlation than the pairwise coevolutionary scores (Table I, Fig. 2(C) vs. Fig. 2(E); Supporting Information Figs. S6–S49 vs. Supporting Information Figs. S11–S14). Subtracting noise from the coevolution scores prior to EVC calculations usually further improved  $R^2$  values [Table I; Fig. 2(D) vs. Fig. 2(F); Supporting Information Figs. S13 and S14). Agreement indicates that network centrality is a robust feature of the coevolutionary data, in contrast to the pairwise scores, which show more variability among algorithms. In addition, we noted that positions with high EVC scores were highly connected to each other within the network of coevolution scores [e.g., Figure 1(A,B)]. However, like the pairwise coevolution scores, EVC connectivity was not obligatorily related to structural proximity (Figs. 3 and 4).

Finally, we considered an ongoing concern about coevolution algorithms—the extent to which the covariation analyses distinguish true “coevolution” from amino acid changes related to phylogeny.<sup>43,67,68</sup> Similar considerations have been raised for the first eigenvector of coevolutionary data.<sup>42,44</sup> The rationale for our approach was that, if phylogeny is contributing a great deal to the covariation/EVC scores, then positions with top scores in phylogenetic algorithms should correlate with those identified by algorithms that are designed to identify amino acid changes that correlate with phylogenetic branches. Several algorithms have been devised to detect this pat-



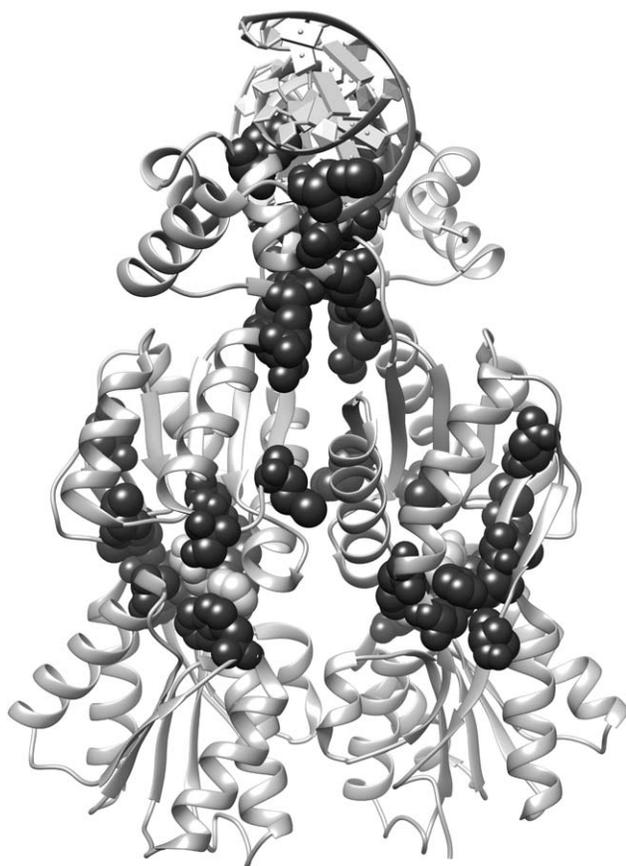
**Figure 3**

Top EVC Positions for Aldolase. This view of aldolase is looking into the active site of one monomer. The top 20 consensus EVC positions (35, 43, 47, 53, 56, 106, 107, 122, 145, 148, 168, 169, 193, 214, 234, 237, 267, 270, 275, and 300; magenta and green spacefilled) of tetrameric human aldolase C are shown on one monomer (PDB: 1xfb<sup>58</sup>). Active site positions are highlighted in dark gray. EVC positions in contact with active site residues are highlighted in green; those without contact are highlighted in magenta.

tern.<sup>1–3,9</sup> Of these, TEA-O<sup>9</sup> is convenient for comparison with coevolution analyses: TEA-O separately scores “specificity” positions that change at the later branches from “conserved” positions that correspond to the primary branches. Due to their conservation, most of the latter are excluded from coevolutionary analyses.

To that end, we compared EVC scores to TEA-O scores for the LacI/GalR and aldolase families. As expected, positions with high TEA-O “conserved” scores show essentially zero correlation with EVC scores for either family (data not shown). For the LacI/GalR family, EVC scores show comparable correlations with TEA-O specificity scores as they do with each other (Table I). However, the aldolase EVC scores show lower correlation with TEA-O (Table I) than they do with each other.

The different correlations may arise because the LacI/GalR family is comprised of paralogs (and thus its MSA could have a stronger phylogenetic signal), whereas the aldolase family is comprised of orthologs. Such differences among protein families could help explain why estimating and correcting for the contributions of phylogeny to intraprotein covariance has been challenging. Further, the presence or absence of phylogeny in the covariation signal does *not* diminish the fact that the top EVC positions are important to structure and function in both LacI/GalR and aldolase families (next section). The possible implications of coexisting and overlapping patterns in evolutionary data are included below, in the Discussion.



**Figure 4**

Top EVC Positions for LacI/GalR. The top 20 consensus EVC positions (17, 27, 29, 51, 52, 55, 57, 98, 102, 117, 125, 150, 157, 160, 161, 193, 220, 291, 293, and 321; spacefilled black) for the LacI/GalR family were mapped onto the homodimeric structure of LacI (PDB: 1efa<sup>53</sup>). The DNA (ribbon at top of structure) and the bound allosteric effector ligand, (orthonitrophenyl- $\beta$ -D-fucopyranoside, spacefilled white) are highlighted to show binding sites.

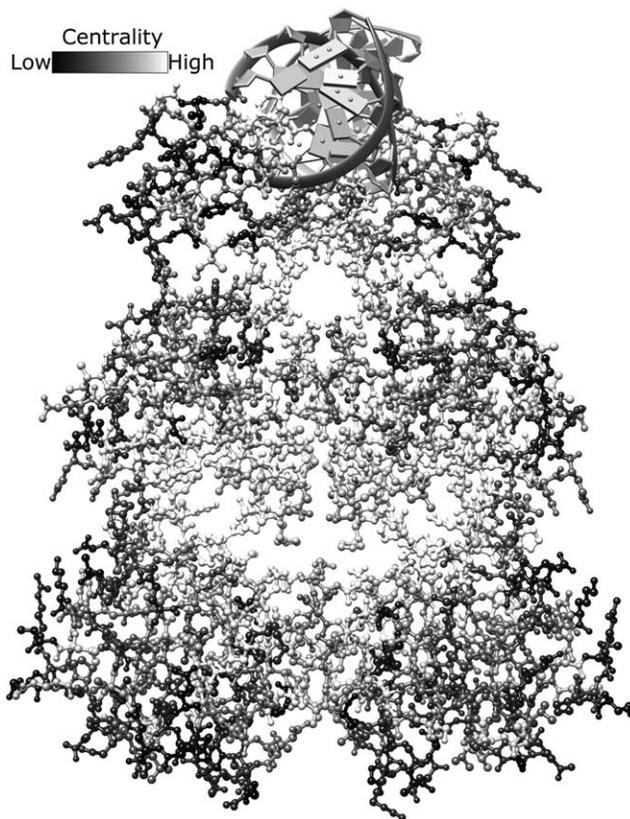
#### Eigenvector central positions are important for structure and function

To assess biological significance, we compared the known structural properties and mutational sensitivities of the top 20 EVC positions. To that end, we first determined a consensus set of positions (see Methods). This was motivated by the fact that, although EVC calculations greatly improved agreement between coevolution algorithms, agreement was never 100%. In choosing this number of sites, we also considered the problem of thresholding. The histograms of EVC scores (diagonals of Supporting Information Figs. S11–S14) show distinct populations of scores, and the top 20 scores fall within the top population. In many cases, the 21st (or lower-ranked) EVC position also fell within this population and might therefore also be functionally important; global analyses that avoid such thresholding are discussed below.

In the aldolase family, the top 20 EVC positions fall near (i) the active site, (ii) the inter-subunit interface or (iii) along the surface on the same face as the ligand binding pocket. (Fig. 3, magenta and green spacefilled) Eight of these positions (35, 106, 107, 145, 148, 270, 275, and 300) contact active site residues and a ninth (position 43) directly participates in catalysis. (Of the eight catalytic residues, six are highly conserved and therefore cannot coevolve). Since these positions fall in crucial functional locations, they are likely to contribute to aldolase function. We speculate that sequence changes at these positions might fine-tune the active site geometry and catalytic parameters to the ecological niche of each organism.

In the LacI/GalR family, the top 20 positions were compared to the extensive structural, mutational and computational studies that have been performed on several paralogs. Eighteen of the top EVC positions are sensitive to mutation in LacI; 19 positions are located in structural regions that are clearly important (Fig. 4; Supporting Information Table S6).<sup>53–57,69–80</sup> Indeed, the top 20 EVC positions reads as an elite list of functionally important positions: in addition to the comprehensive LacI mutagenesis study,<sup>70,79,80</sup> many of these positions were individually targeted for mutagenesis after in-depth structure/function analyses and molecular dynamics simulations suggested (and experiments confirmed) their importance. The two positions that lack known mutational response (27 and 102) have only been mutated in LacI. These positions might be key in other LacI/GalR proteins; we recently demonstrated that mutating non-conserved positions can have widely different outcomes among homologs.<sup>77</sup>

As we noted above, setting an arbitrary threshold for top EVC positions might miss important information about other positions. To consider the full range, we color-coded each all amino acids in each structure, according to their rank-ordered EVC scores. Given the continuum of shades that represents the range of scores (Fig. 5, Supporting Information Figs. S15 and S16 color bars), it is striking that structural regions emerge that are dominated by one color. For the LacI/GalR family, positions assigned high EVC scores cluster in the interior of the protein (Fig. 5, white and Supporting Information Fig. S16, magenta), especially near the binding sites of the allosteric effector ligand and DNA operator and along the intermonomer interface. In the aldolase family, the top 20 consensus positions were representative of the global pattern: positions with higher centrality scores were generally found on the protein interior [Supporting Information Fig. S15(D)], at the tetramerization interaction surface [Supporting Information Fig. S15(C)], and on the same surface of the protein as the active site [Supporting Information Fig. S15(A)]. In contrast, the opposite surface—far from the active site—generally had lower EVC scores [Supporting Information Fig. S15(B)].



**Figure 5**

Global structural analysis LacI/GalR EVC scores. The homodimeric structure of LacI (PDB: 1efa<sup>53</sup>) is color-coded based on the rank order of each positions' EVC score (high scores, white; low scores black). Bound DNA is shown at the top of the figure in gray. Inducer ligands bind in the central pocket of each monomer. Given the range of available scores (color bar at the top of the figure), it is striking that some structural regions are dominated by similar scores. For example, the inducer binding site is surrounded by positions with high EVC scores (very light gray and white). This figure is shown in color in Supporting Information Figure S16.

## DISCUSSION

Pairwise coevolution of amino acid positions provides an incomplete view of protein evolution. Science's common understanding of protein structure/function relationships suggests that *multiple* amino acids should be constrained together during evolution. Direct calculations for detecting larger groups of coevolving positions have been limited by the sheer number of output scores, which increase exponentially. (For a protein with  $N$  positions, pairwise coevolution calculates  $N^2$  scores; three-way coevolution would calculate  $N^3$  scores, and so forth) Thus, investigators have turned to analyses of pairwise data to identify multiply constrained positions.<sup>25,42,62,81–83</sup> For one implementation of SCA, spectral decomposition of pairwise scores was used to determine the first eigenvector, but results were found to be heavily influenced by the sequence conservation for each position.<sup>84</sup> Here, we have avoided such spurious signals via the use of shuffled alignments to estimate and subtract non-coevolutionary signals. Our results show

that subtracted EVC scores contain information about amino acid positions key to structure and/or function of the LacI/GalR and aldolase families.

Strikingly, our analyses reconciled the contradictory results from alternative coevolution algorithms, showing that this deeper signal of evolutionary change is robust. Other strengths of EVC calculations are that they (i) are mathematically guaranteed to have a unique top eigenvalue,<sup>52</sup> (ii) provide a tractable means to detect positions that are constrained by multiple partners, and (iii) defer problems introduced by thresholding until the final analyses. Positions with large EVC scores appear to largely follow the pattern that can be seen qualitatively in Figure 1(B): some positions must simultaneously reconcile coevolutionary constraints from many other positions. For these positions, no single constraint is strong, but many are moderate [Fig. 1(B)]. This contrasts with the case illustrated in Figure 1(C): when two positions have strong coevolutionary constraints between them, this can override the influence of most other positions (which subsequently manifests as low pairwise scores).

Despite the increased correlation over pairwise data, the EVC correlations do not perfectly reconcile the alternative algorithms. First, for aldolase, the correlations between SCA and other algorithms are lower ( $R^2 < 0.5$ ) than the other comparisons. Perhaps SCA (i) is not an appropriate analysis for this family or (ii) detects a different evolutionary signal for this family than the other algorithms. Second, for the LacI/GalR family, half of the EVC algorithm correlations had  $R^2$  values less than 0.5 (Table I), whereas those for aldolase (except SCA) were generally above 0.7. We speculate that this is related to the fact that the LacI/GalR family comprises paralogs, whereas the aldolase MSA contains only orthologous sequences. The lower LacI/GalR EVC correlations were not explained by the range of sequence identities in its MSA, since the aldolase MSA spans a comparable range (Supporting Information Table S1). Nevertheless, the EVC correlations were a significant improvement over pairwise correlations; every prior step of the calculations had much worse correlations.

In addition, we wish to stress that our results do not imply that EVC positions are a priori more important to structure or function than positions with other types of evolutionary patterns. Instead, we propose that protein evolution should be thought of as having many types of constraints that affect different groups of positions, some of which is captured by pairwise coevolution and others by EVC calculations. Still other patterns include highly conserved positions<sup>10</sup> and changes that track with phylogeny.<sup>1–3,9</sup> We have also observed that, as one common scaffold evolves functional diversity, the positions under evolutionary pressure can move to different locations.<sup>28</sup>

To date, all of these evolutionary patterns appear to identify mutationally sensitive positions, and there is no reason to constrain “importance” to just one pattern of change. Indeed, our previous studies show that a large number of positions, representing a variety of evolutionary patterns, contribute to the evolution of altered LacI/GalR protein function.<sup>28,32,77</sup> We expect similar results for other protein families. Even families that comprise a single ortholog—such as aldolase—likely evolve functional variation that is appropriate for distinct biological niches. (In the case of aldolase, the central metabolism of each organism must adapt to variations in nutrient availability.)

Finally, some positions might be described by multiple evolutionary patterns. One possibility (although not dominant in the LacI/GalR or aldolase families) is that positions with top pairwise scores are also top EVC scores. Another possible overlap is between positions with strong coevolution and phylogenetic scores: the fact that covariation can occur from either coevolution or phylogeny is usually seen as a drawback to these calculations, but the changes of some coevolving pairs might also track with phylogeny. The same is true for EVC positions, as may be the case for the LacI/GalR family.

Nor does the overlap diminish the functional significance of positions with top EVC scores, as shown by their sensitivity to mutagenesis and/or their key structural locations. Finally, the aldolase data show that EVC scores are not always dominated by phylogeny, because EVC and TEA-O scores show lower correlations than the EVC scores with each other (Table I). As we noted above, each protein family may have different phylogenetic contributions to covariation scores; a strong difference could occur between families that comprise orthologs versus those comprising paralogs.

One remaining question is whether positions with different types of evolutionary patterns have different mutational responses. Conserved positions almost always exhibit what we recently called “toggle” behavior<sup>77</sup>—at a given position, most amino acid substitutions abolish function; furthermore, compensatory changes at other positions are seldom identified. Mutational results for positions with high pairwise coevolutionary scores have been harder to predict.<sup>77</sup> Supporting Information Figure S10 shows that many of the top pairwise positions in LacI/GalR also act as functional toggles when mutated individually. For coevolving pairs of positions, early expectations were that their mutations would compensate each other. However, results from double mutant cycles have not shown clear-cut patterns; some show no linkage between coevolving pairs of positions,<sup>62</sup> others show epistasis (nonadditivity), with the caveats that nonadditivity was not always predictable or uniquely limited to the coevolving positions.<sup>85–87</sup>

Given the high prevalence of epistasis that has been documented for changes at nonconserved positions during evolution,<sup>88–91</sup> we expect that EVC positions will also show nonadditivity for combinatorial mutations. Indeed, for the LacI/GalR family, mutational epistasis has already been documented among several of the top 20 EVC positions.<sup>92</sup> Furthermore, multiple substitutions at three top EVC positions in LacI/GalR (51, 52, and 55) reveal a “rheostat” behavior: that is, the multiple variants for one position could be rank-ordered to show a progressive effect on function that spanned orders of magnitude. It is intriguing to consider (i) whether the strong edges between high EVC positions can predict which groups of positions will show nonadditivity with each other; or (ii) whether positions that have both strong EVC and strong phylogeny scores (such as in LacI/GalR) have different mutational outcomes than positions that have either strong EVC or strong phylogenetic scores (such as in aldolase).

In conclusion, EVC network centrality detects positions that can be important to protein structure and function. Furthermore, EVC calculations are more consistent between algorithms than pairwise coevolution scores, indicating that these central nodes are a robust property of coevolution networks.

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